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MATDIP301

Third Semester B.E. Degree Examination, Feb./Mar. 2022
Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Express the complex number $\frac{2-i\sqrt{3}}{\sqrt{3}+i}$ in the form of x + iy. **(06 Marks)**

b. Express $\sqrt{3}+i$ in the polar form and hence find modulus and amplitude. **(07 Marks)**

c. Prove that $\left(\frac{\cos\theta+i\sin\theta}{\sin\theta+i\cos\theta}\right)^4=\cos8\theta+i\sin8\theta$. **(07 Marks)**

2 a. Find the n^{th} derivative of $e^{ax}\sin(bx+c)$. **(06 Marks)**

b. If $y=\sin^{-1}x$ then prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$. **(07 Marks)**

c. Find the n^{th} derivative of $\frac{x+3}{(x+2)(2x+3)}$. **(07 Marks)**

3 a. Find the angle between the curves $r=\sin\theta+\cos\theta$ and $r=2\sin\theta$. **(06 Marks)**

b. Find the pedal equation for the curve $r^2=a^2\cos2\theta$. **(07 Marks)**

c. Using Maclaurin's series expand $y=\tan x$ upto the term containing x^5. **(07 Marks)**

4 a. If $u=\sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\tan u$. **(06 Marks)**

b. If u = f(xz, y/z), prove that $x\frac{\partial u}{\partial x}-y\frac{\partial u}{\partial y}-z\frac{\partial u}{\partial z}=0$. **(07 Marks)**

c. If $x+y+z=u$, $y+z=v$ and $z=uvw$ then find the value of $\frac{\partial(x,y,z)}{\partial(u,v,w)}$. **(07 Marks)**

5 a. Obtain the reduction formula for $\int\sin^n x\,dx$, where n is a positive integer. **(06 Marks)**

b. Evaluate $\int_0^{\pi/2}\sin^3\theta\cos^7\theta\,d\theta$. **(07 Marks)**

c. Evaluate $\int_0^a\int_0^{\sqrt{a^2-x^2}}x^2y\,dy\,dx$. **(07 Marks)**

6 a. Evaluate $\int_0^1\int_0^2\int_0^3 xyz\,dz\,dy\,dx$. **(06 Marks)**

b. Define Beta function and prove that $\beta(m,n)=\beta(n,m)$. **(07 Marks)**

c. Using beta and gamma functions evaluate $\int_0^{\pi/2}\sin^4\theta\cos^5\theta\,d\theta$. **(07 Marks)**

7 a. Solve $\frac{dy}{dx} = \frac{y}{x} + \sin\frac{y}{x}$. **(06 Marks)**

b. Solve $(1+y^2)dx = (\tan^{-1}y - x)dx$. **(07 Marks)**

c. Solve $(1+e^{x/y})dx + e^{x/y}(1-x/y)dy = 0$. **(07 Marks)**

8 a. Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} + 4$. **(06 Marks)**

b. Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x x^2$.

c. Solve $(D^2 + D + 1)y = x^2 + x + 1$. **(07 Marks)**

(07 Marks)

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